THE CAPTURE THEORY OF COSMICAL EVOLUTION CONFIRMED BY THE LATEST RESEARCHES ON THE ORIGIN OF STAR CLUSTERS.

Introductory Remarks.

It is a very remarkable fact that since the epoch of Sir William Herschel little serious consideration has been given to the origin of star clusters, until within the last few years. Accordingly the thoughtful suggestions thrown out by that unrivaled man long proved largely if not entirely barren of fruitful results for the development of a science of cosmogony, because his early ideas were lost sight of or forgotten. We have labored under a strange delusion, of preferring the theories of Laplace to those of Herschel, but have at last found the way from darkness to light, from error to truth. And it happens that of all the investigations yet made in cosmogony those relating to the neglected subject of star clusters are the most convincing and least open to objection.

In a paper entitled "Dynamical Theory of the Globular Clusters and of the Clustering Power Inferred by Herschel from the Observed Figures of Sidereal Systems of High Order," recently communicated to the American Philosophical Society held at Philadelphia, and just published in the *Proceedings* of that illustrious society, I have examined the whole problem of the origin of clusters in a somewhat exhaustive manner. By the use of mathematical methods of rigorous character, I was able to develop the most convincing proofs that these aggregations of stars have arisen by the process of capture in the course of millions of ages. It will be the main purpose of this article to discuss the results arrived at in this general investigation of the origin of star clusters; but before taking up this subject in detail it will be allowable to treat briefly of the conclusions reached by the illustrious Poincaré in his *Leçons sur les Hypothèses Cosmogoniques,* 1911, and to notice also the unimportant objections advanced by Prof. Charles André in *Scientia,* No. 2, 1912.

The Views of Poincaré.

In his *Leçons sur les Hypothèses Cosmogoniques,* 1911, M. Poincaré gives a summary of the various theories of cosmogony, and in two chapters discusses results arrived at in my *Researches on the Evolution of the Stellar Systems,* Vol. II, 1910.* He exam-

* These chapters appeared in translation in *The Monist* of July, 1912.

ines and adopts the proof therein given that the roundness of the orbits of the planets and satellites is due to the secular effects of the action of a resisting medium. He concurs in the capture of satellites, as well as in the new theory of spiral and ring nebulae. M. Poincaré especially remarks how well the resisting medium explains the roundness of the orbits of the planets and satellites; and altogether is favorable to the recent development of cosmogony into a new science of the starry heavens.

One point in my work, however, has been slightly misunderstood by Poincaré, and I will therefore dwell upon it here. After outlining the leading principles of the Capture Theory he adds that while it fully explains the roundness of the orbits, it does not give a satisfactory explanation of the inclinations of the planetary orbits. But here he has evidently lost sight of the nature of the spiral nebula from which our solar system is supposed to have arisen.

The Capture Theory means primarily that all revolving bodies are added on from without,—the planets being added to the sun and the satellites added to the several planets; but it is not held that the entrance into our system was from all directions over the celestial sphere and thus entirely at random. On the contrary it is carefully explained in my work that the system from the beginning had a fundamental plane of maximum areas, due to the fact that a spiral nebula is formed by two principal streams coiling about one another; and the plane of maximum areas is the plane determined by these predominant streams.

In the condensation of a nebula an infinite number of minor streams probably are involved; but the whirling motion is made possible only by two predominant streams, as shown in photographs of spiral nebulae. That is to say, there is more matter in the two large streams than in the smaller ones; and this gives a fundamental plane to the system when it becomes mature, just like that in which the planets of the solar system are found to move. The planetary orbits ought not to lie exactly in the same plane, but near an invariable plane, such as Laplace in 1784 proved to exist in every system of bodies subjected to the mutual gravitation of its parts.

The comets or smaller masses of nebulosity naturally should be inclined at all angles to the invariable plane; but as they intersect that plane twice in their orbital motion about the sun, they will sooner or later pass near a planet revolving in an orbit lying near the fundamental plane of the system, and their orbits are thus subjected to profound changes of position as well as of form and extent. Count-

less comets are destroyed in building up the planets; so that much matter not originally lying in the plane of the planets is finally captured and drawn into that plane, as the masses of the planets gradually augment, and they are drawn nearer the sun in orbits becoming ever smaller and smaller, and rounder and rounder.

Thus on the one hand the resistance of cometary matter reduces the size of the planetary orbits and makes them rounder, while on the other hand the growth of these masses increases their mutual attraction; and if they were originally near the plane of the predominant streams, in time they come to move almost exactly in one plane, as now observed in our actual planetary system.

For as our system has shrunk, and the original orbits were hundreds of times larger than at present, the nuclei at the outset were not necessarily very near the plane in which they now move, but may have departed from it considerably. Mutual inclinations of a few degrees now found in our system are magnified at hundredfold primordial distances into very great absolute distances; so that the original streams need not have been at all compressed, but may have been exceedingly diffuse, just as actual nebulae appear to be.

Accordingly it is a remarkable fact that the theory which accounts for the roundness of the orbits of the planets also explains the small mutual inclinations of their orbits, and the rotation of the sun about an axis nearly perpendicular to the plane in which the planets revolve. The explanation of the origin of our system from a spiral nebula thus appears to be entirely satisfactory.

The Views and Objections of André.

The objections to the Capture Theory advanced by André are easily shown to be without the slightest foundation. It is quite unnecessary to consider most of them, and I will therefore content myself with the three chief ones, which will sufficiently show the weakness of the rest.

1. André claims that the spherical expansion in Babinet's criterion as I have used it is not strictly in accordance with Laplace's theory, because Laplace did not imagine the sun's atmosphere to be expanded in a spherical form, but rather in the form of a flat disc. This objection is quite devoid of foundation, as will appear from the following simple considerations.

a. If the expansion be spheroidal, as a flat disc, more of the matter is at greater distance from the center, for given volume, than in a spherical expansion; so that the moment of inertia is in-

creased, and with constant moment of momentum the angular velocity is therefore *decreased.* Hence a discoidal expansion of the sun is more unfavorable to Laplace's hypothesis than the spherical expansion used by me. For in case of a sphere the moment of inertia is shown in works on the calculus to be $2/5(Mr^2)$, where r is the radius and M the mass; in an ellipsoid with equatorial axes a and b it is $M/5(a^2+b^2)$, and when $a=b$, as in an ellipsoid of revolution, this becomes $2/5(Ma^2)$, a being the equatorial axis.

b. To reduce this to numbers I took ellipsoids with meridian sections of eccentricity 0.10, 0.25, 0.5, and 0.8, giving oblatenesses of 0.00501, 0.03176, 0.13397, and 0.40000 respectively; and found $a^2=1.00336r^2$; $a^2=1.0217r^2$; $a^2=1.1525r^2$; $a^2=1.4057r^2$. *This shows how the moment of inertia increases as the oblateness increases, and thus proves a corresponding decrease of the angular velocity of rotation below that published in my tables of Babinet's criterion.* The objection of André therefore has not the slightest foundation, because my calculations are more favorable to Laplace's theory than those based on the theory of an oblate spheroid.

2. André dwells on the fact that Laplace imagined only *the atmosphere* of the sun expanded to the orbits of the planets. But as the sun itself when so expanded becomes much rarer than most atmospheres we are familiar with, it is readily seen that this point is not well taken. When the sun is expanded to Neptune's orbit, the average density of the nebula is 260 million times less than that of air at sea level. Nothing more need be said on this point. Such a medium could exert little or no hydrostatic pressure from the center, and Laplace's theory of the detachment of zones of vapor under conditions of hydrostatic pressure implies that he overlooked the rarity of this medium, which makes such a thing as hydrostatic pressure quite impossible. No alteration of central arrangement of density would materially change this result, and we may thus dismiss it without further comment.

3. As the centrifugal force, by Babinet's criterion, is only a ten millionth part of that required to detach the earth, and a three hundred millionth of that required to detach Neptune, while the hydrostatic pressure likewise is insensible, it is clear that no such detachment as Laplace imagined ever took place. André, Ligondes and other French writers are simply injuring the memory of Laplace by presenting to the Paris Academy of Sciences conclusions which would be immediately rejected by Laplace himself if he were living to-day.

After having studied the works of this great master of celestial mechanics from the days of my youth, I believe I have followed his spirit in rejecting what is now known to be false. Professor André is in the unfortunate position of having written books favorable to the abandoned theory of Laplace; but he should aim at truth rather than perpetual consistency, and modify his views to meet the latest discoveries in science. For a true philosopher does not aim at supporting his earlier writings, but at gradually attaining the truth, even if his first work has to be modified or entirely abandoned. The successors of Laplace obviously should act upon this laudable principle.

4. Even if the retrograde satellites and a multitude of other phenomena did not tell us unmistakably that all the satellites have been captured, and we still tried to explain these bodies by the detachment theory of Laplace, we should remain quite in the dark as to the origin of the observed rotations. They would be simply assumed, and not explained; and so we should have no rational theory of the formation of the solar system; whereas the Capture Theory gives a simple and natural explanation of the rotations and obliquities as well as the orbital motion of the satellites, and the variations of their brightness, the lunar craters and maria and kindred phenomena; and all the phenomena are so woven together that it is impossible to doubt the truth of the new theory.

In the same way, even if the solar nebula could have rotated rapidly enough to detach zones of vapor as Laplace imagined, it would still be impossible to account for so rapid a rotation. Fortunately Babinet's criterion shows that no such rapid rotation for the detachment of zones of vapor ever took place; and that Laplace was deceived by the roundness of the planetary orbits, which we now recognize to be due to the secular action of the nebular resisting medium formerly pervading our solar system.

Necessity for Wider View of all Sidereal Systems.

It requires no elaborate argument to convince any philosophic investigator that the laws of cosmical evolution can best be deduced from the study of nature in the widest sense. The narrowness of the cosmogony of Laplace arose from the fact that it was based wholly on our solar system, and that too before the system was fully understood. The roundness of the orbits of the planets and satellites and the survival of a ring about Saturn led to the idea

that all these bodies had originated by the detachment of rings. Yet as soon as the orbits of the double stars were determined, they were found to have eccentricities of every degree, between the round orbits characteristic of the planets and satellites and the very elongated orbits characteristic of the comets. The development of double stars obviously could not have been by the formation of rings as imagined by Laplace.

Accordingly without such a comprehensive view of the different types of systems it would be vain to hope for the deduction of a general law of nature. The folly of adhering to the old methods of Laplace based on an imperfect knowledge of the solar system alone is thus apparent; and after what is now shown, from Babinet's criterion, as to the impossibility of detaching masses or rings, there is no course open to us but to reject Laplace's hypothesis once for all. It does not give us a general law of nature, and is not true even for the special case of the solar system.

Our hope for finding the law of nature must be based on the study of double and multiple stars, and sidereal systems of higher order. Now it happens that of the various sidereal systems known to the astronomer, the globular clusters are the most complex, and at the same time the most symmetrical and regular in their constitution. If therefore any light can be obtained on the formation of sidereal systems of such high order, it might be possible to derive principles which could be applied to less symmetrical systems of lower order. This is what I have done in my recent investigation of the origin of clusters. Having deduced the law of nature from the highest and most complex systems, with wonderful regularity of figure, I have proceeded to apply it also to systems of the lowest type, as the solar system and the double and multiple stars. This new method of procedure is so important, that it becomes advisable to explain it in some detail.

Nature of Clusters, Average Distance of the Stars Apart, Increase of Density Towards Center.

Sir William Herschel always considered the globular clusters to be the most wonderful of all sidereal systems. He never ceased to marvel at the existence of these swarms of stars, which were known to be aggregations of suns; and he inferred that at length they had been moulded into the spherical form by the action of central powers.

Even in the time of Herschel it was recognized that the clusters are very far from the earth, and thus that the component stars are not really close together, but separated by intervals which are very great compared to those which separate the planets from the sun.

More modern discovery has confirmed the sagacious conjectures of the great Herschel. The latest investigation of the profundity of the Milky Way, which I finished in November, 1911, and have just published in the *Proceedings* of the American Philosophical Society at Philadelphia, shows that the remotest clusters are removed from us by at least a million light-years. Indeed this determination of the depth of the Milky Way shows that the remotest stars may be removed from us by distances of five or ten million light-years; but even with most of the clusters at distances of hundreds of thousands of light-years, it is possible to say with certainty that the average space between the stars in globular clusters is of the order of a light-year, which is 63275 times the distance of the earth from the sun. We thus have the spectacle of systems of stars separated by great intervals, but so remote as to be drawn together by perspective into a small angular space on the surface of the sky.

The density in these masses of stars was found by Herschel to be always greatest towards the center; and in fact to be in excess of that corresponding to the supposition of equal scattering. Herschel therefore inferred that the accumulation in the centers of the clusters must be due to the secular action of a clustering power, which he believed to be nothing else than universal gravitation working over millions of ages. He remarked that the Milky Way presented the aspect of a clustering stream traversing the heavens as an irregular band of milky light; and as he had found the sidereal universe to be greatly extended in the direction of the plane of the Milky Way, he correctly inferred that the clustering stream thus presented to the eye was the effect of distance and of local aggregations of the stars into star-clouds and clusters. The stars are spread out into a comparatively thin stratum, and at great distance the effect is to give the appearance of the Milky Way, which thus appears as a clustering stream several degrees in width.

How the Stars are Captured in Clusters.

In the memoir above referred to I have established the capture of stars by a cluster, and the secular shrinkage of the cluster, by the use of Green's theorem for the transformation of a triple in-

tegral appropriate for space into a double integral over the surface of the cluster. By showing that the surface shrinks as the result of close appulses among the stars, and also as the outcome of mutual gravitation, even when no close approach occurs, it is found that the cluster becomes more and more compressed, with density accumulating towards the center.

The attraction of members of a cluster is analogous to surface tension in working to decrease the volume of a bubble, or in rounding up a drop of dew, to give minimal surface for a given volume. In the same way gravity tends to make a planet perfectly round, except as modified by rotation into an oblate figure. Herschel used such analogies in his argument for a clustering power, which he inferred to be moulding the figures of clusters. And recently I have tested his suggestion mathematically, and found a conclusive proof that the argument is correct.

To give a simple analogy for the capture of stars in clusters, with known processes in the solar system, we may remark that Jupiter captures the comets crossing over his orbit, and transforms their paths till they lie wholly within that of the planet. In this way he has captured quite a family of comets and thrown their orbits within his own orbit. Now in the memoir above referred to I have shown that a *shell of stars* in a cluster acts very much as Jupiter does on the comets—and thus tends to reduce the path of an oscillating star till it comes within the confines of the shell.

Accordingly if a star from without once enters a cluster, and thus begins to traverse the series of shells of which the cluster is made up, it will never quit the swarm but be gradually drawn in, and captured, during one or more complete oscillations. The extent of its outward journey from the cluster, if any occurs, will be decreased, until finally it is dragged down to the level of the shell, and becomes a member of the cluster. This is one of the most remarkable results of our dynamical theory of clusters. The Capture Theory being thus verified for these globular masses of stars, it naturally may be expected to operate in systems of lower order.

No Possible Origin of Clusters Except that Outlined by the Capture Theory.

The globular clusters are so perfectly symmetrical that they become of high interest in elucidating the problems of cosmogony. For it is not conceivable that systems of such large mass, great

extent and perfect symmetry, can have arisen, except by the gathering together of stars from a wider extent of space.

No process of collision, for example, would account for the globular clusters; for by impact the matter of two hypothetically disrupted masses would neither be symmetrically distributed nor dispersed over such a vast space as that now occupied by the thousands of suns composing a cluster. Then, again, to be effective such hypothetical collision would have to be between approximately equal giant suns; and there are too few stars of such enormous mass for pairs of them ever to come into bodily collision.

Accordingly, a little consideration shows us, on the one hand, that such giant collisions would not occur; and, on the other, that even if they could take place such widely diffused and symmetrical swarms of stars could not arise by this process. The globular clusters therefore are due to the aggregation of stars once symmetrically and widely distributed in space. This gives us a good illustration of the Capture Theory on the most stupendous scale. Similar views were reached by Herschel, without mathematical investigation of the subject, such as I have recently developed; and it may be remarked that he found the evidence of a clustering power most convincing.

The *New General Catalogue of Nebulae and Clusters,* published by the Royal Astronomical Society of London in 1888, contains a list of more than one hundred globular clusters, mostly distributed along the course of the Milky Way. The clustering of the stars into great systems about so many centers shows how general and widespread this tendency is in nature.

If now we recall that only the oldest sidereal systems can have attained a state of perfect symmetry, it is obvious that a larger number of sidereal systems might be expected to have an irregular and unsymmetrical aspect. The globular clusters are therefore only a part of the aggregations of stars exhibiting the effect of the clustering power; but the perfection of this type of system renders it eminently adapted to disclosing the process by which all clusters are formed. For if the law of nature can be deduced from the perfect type of sidereal development, it may with equal certainty be inferred to operate in those sidereal systems which have not yet attained to full maturity. By investigating the different types of sidereal systems our studies may thus disclose the general law of cosmical evolution and embrace phenomena extending over millions of ages!

The Law of Nature Embraces also Systems of Lower Order, and Therefore the Planetary System and the Systems of Double and Multiple Stars.

Those who believe in the uniformity and continuity of the laws of nature, as laid down by Newton in the *Principia,* 1687, will quickly realize that the law of cosmical evolution established for the globular clusters should necessarily hold also for systems of lower order. Rule I: "We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances." Rule II: "Therefore to the same natural effects we must, as far as possible, assign the same causes."

Accordingly, in line with these rules of Newton, I have shown that the Capture Theory will explain the formation of the solar system, as well as the double and multiple stars: and having found the principle to be the same throughout the sidereal universe, I have inferred that nature's law everywhere is one of adding on from without. The component stars are added to the clusters, and drawn nearer and nearer the center; the planets added to the sun and made to revolve in smaller and smaller and rounder and rounder orbits. Likewise the satellites were added on to their several planets, and the moon captured by the earth. The double and multiple stars were formed on the same principle—the nuclei having originated in the distance, and subsequently approached the centers about which they now revolve. This gives us a general law of nature of the utmost simplicity.

And not only is the generality of the law proved by force of analogy, but also by direct mathematical demonstrations in the solar system, deduced from Babinet's Criterion; while in the clusters the proof is so obvious that it need scarcely be emphasized. The demonstration of this law in the double and multiple stars is similar to that available in the solar system; and moreover is supported by the analogy of the clusters, into which the multiple stars merge by insensible degrees, when the number of bodies in a group is increased indefinitely.

Accordingly nothing is more certain than that the law of cosmical evolution now recognized is the true law of nature. It does not even resemble the abandoned theory of Laplace, but has considerable resemblance to the general outline of the nebular hypothesis as traced by Sir William Herschel. In particular Herschel's theory of clusters, as originating by the aggregation of isolated

stars is deserving of attention; for this is the earliest outline of a process of capture similar to that now worked out in detail and shown to be applicable to all types of systems observed in the sidereal universe.

In conclusion it seems advisable to close this discussion by the following summary quoted from my latest memoir on *The Dynamical Theory of Clusters.*

Summary and Conclusions.

Without attempting, in this closing section, to recapitulate the contents of this memoir in detail, it may yet be well to draw attention to some of the most significant conclusions at which we have arrived.

1. As intimated in the first section of this paper the problem of n-bodies, under ideal dynamical conditions, remains forever beyond the power of the most general methods of analysis; but the dynamical theory of clusters gives us the one secular solution of this problem found under actual conditions in nature. For when n is of the order of 1000, so as to give rise to a cluster, the clustering power observed by Herschel operates to exhaust the mutual potential energy of the system, and bring about increasing accumulation in the center, so that the cluster finally unites into a single mass of enormous magnitude. Probably the giant stars of the type of Canopus and Arcturus have arisen in this way.

2. And since attendant bodies of every class—as satellites, planets, comets, double and multiple stars—tend everywhere to approach the centers about which they revolve, as an inevitable effect of the growth of the central masses and of the action of the resisting medium over long ages, it follows that the secular solution of the problem of clusters is more or less valid for all cosmical systems. They finally end by the absorption of the attendant bodies in the central masses which now govern their motions.

3. The dynamical theory of globular clusters shows that the clustering power inferred by Herschel is nothing else than the action of universal gravitation; and that it operates on all sidereal systems, but does not produce the cumulative effect which Herschel ascribed to the ravages of time inside of millions of ages.

4. The globular clusters are formed by the gathering together of stars and elements of nebulosity from all directions in space; and this points to the expulsion of dust from the stars of the Milky Way,

and its collection about the region of the formation in such manner as to give essential symmetry in the final arrangement of the cluster, which doubtless has some motion of rotation, and originally a tendency to spiral movement.

5. The stars and smaller masses are captured by the mutual action of the other members of the cluster, and worked down towards the center of the mass. This gives a central density in excess of that appropriate to a sphere of monatomic gas in convective equilibrium (A. N. 4053 and A. N. 4104).

6. The density of the clusters is greater on the outer border than in a globe of monatomic gases, which shows that stars are still collecting from the surrounding regions of space. The starless aspect of the remoter regions about clusters is an effect of the ravages of time, as correctly inferred by Herschel in the course of his penetrating sweeps of the starry heavens.

7. And just as clusters under the mutual gravitation of the component stars contract their dimensions, with time, chiefly owing to the growth of the central masses, so also do other systems, whether the mass-distribution be *single,* giving a system made up of a sun and planets, or *double, triple* and *multiple,* giving binary, triple or multiple stars, or sidereal systems of still higher order. The tendency everywhere is from a wider to a narrower distribution of the large bodies; while the only throwing off that ever occurs is of particles driven away from the stars by the action of repulsive forces.

8. The orbits of the stellar and planetary systems are decreased by the growth of the central masses and rounded up by the action of the nebular resisting medium. And in like manner all clusters tend to assume spherical or globular figures, so as to justify the expression of Plato, that the Deity always geometrizes; or Newton's remark that the agency operating in the construction of the solar system was "very well skilled in mechanics and geometry."

9. Newton required the intervention of the Deity to give the planets revolving motion in their orbits, because in the absence of repulsive forces he could not account for the dispersion of the matter so as to produce the tangential motions actually observed. By means of the theory of repulsive forces, however, it is now possible to explain these prôjectile motions, which Herschel likewise pointed to as the chief agency for the preservation of sidereal systems. The only assumption necessary is an unsymmetrical figure of the primordial nebula, giving a whirling motion about the center as the system develops; and since the dust gathers from all directions it is certain

that this lack of perfect symmetry will always develop, as we see also by the spiral nebulae.

10. It is this unsymmetrical form of the spiral nebulae produced by the gathering of the dust from the stars, or the slight relative tangential motion of stars formed separately but finally made to revolve together as a binary system, that gives the projectile forces with which they are set revolving in their orbits. In no case have they resulted from the rupture of a rotating mass of fluid under conditions of hydrostatic pressure as formerly believed by Darwin, Poincaré and See.

11. Even if the rotation could become rapid enough to produce a separation, under conditions of hydrostatic pressure, by rupture of a figure of equilibrium, there would still be the equal or greater difficulty of explaining the origin of the primitive rapid rotation. This last difficulty escaped notice till we came to assign the cause of rotations, and found that mechanical throwing off was impossible under actual conditions in nature. It is therefore recognized, from the definite proof furnished by Babinet's criterion in the solar system, that such a thing as a throwing off never takes place; but that all planetary and stellar bodies are formed in the distance, and afterwards near the centers about which they subsequently revolve.

12. This gives us a fundamental law of the firmament—the planets being added on to the sun, the satellites added on to their planets, the moon added on to the earth, and the companions added on to the double and multiple stars—which now is found to be beautifully confirmed by the dynamical theory of the globular clusters. *It is not often that such a great law of nature can be brought to light, and it is worthy of the more consideration from the circumstance that it explains all classes of stellar systems by a single general principle.*

13. As sidereal systems of lower order are conserved by projectile forces, it is probable that the clusters likewise have a spiral motion of rotation, with similar projectile forces tending to counteract simple progressive collapse. The period of the orbital revolution of the stars of a cluster is found to be common to all, without regard to the dimensions of the elliptical orbits described; and thus the whole system may have a common period of oscillation, after which the initial condition is perfectly restored. This possibility in the dynamics of a cluster is exceedingly wonderful and results from the central attraction depending directly on the distance.

14. The equality of brightness in star clusters shows that some process of compensation between the attractive and repulsive forces

has produced stars of wonderful uniformity of luster. Thus the present investigation confirms the previous researches on the evolution of the stellar systems, which have laid the foundations for a New Science of the Starry Heavens.

15. Accordingly the Capture Theory of cosmical evolution being now firmly established for the clusters, where the nature of the process is entirely clear, it becomes at once a guide to us in dealing with systems of lower order; and we see that the law of nature is uniform and everywhere the same, the large bodies working in towards the centers of attraction, while the only throwing off that ever takes place is of small particles driven out of the stars by the action of repulsive forces. All planetary bodies are formed in the distance, and have their orbits reduced in size by increase of the central masses, and rounded up by moving in a resisting medium. *This is a perfectly general law of the sidereal universe. It verifies the early conjectures of Plato and Newton concerning the stability of the order of the world and shows that these illustrious philosophers were quite justified in concluding that the Deity always geometrizes.* The spiral nebulae tend to develop systems with rounder and rounder orbits, and the clusters made up of thousands of stars assume globular figures with minimal surfaces and internal density so arranged as to give maximum exhaustion of the potential energy.

16. This is geometry of the most marvelous kind, as we find it impressed on the systems of the sidereal universe; and the perfection of this most beautiful science of celestial geometry may be considered the ultimate object of the labors of the astronomer. The philosophic observer is not and never can be content with mere observations of details which do not disclose the living, all-pervading spirit of nature.

17. If, then, the mystery of the gathering of stars into clusters is now penetrated and traced to the clustering power of universal gravitation, so also is the mystery of the *converse problem of starless space,* which was a subject of such profound meditation by the great Herschel.

18. This incomparable astronomer likewise correctly concluded that the breaking up of the Milky Way into a clustering stream is an inevitable effect of the ravages of time; but we are now enabled to foresee the restorative process, under the repulsive forces of nature, by which new nebulae, clusters and sidereal systems of high order will eventually develop in the present depopulated regions of starless space.

19. If there be an incessant expulsion of dust from the stars to form the nebulae, with the condensation of the nebulae into stars and stellar systems, while the gathering of stars drawn together by a clustering power operating over millions of ages gives at length a globular mass of thousands of stars accumulating to a perfect blaze of starlight in the center, but surrounded externally by a desert of starless space resulting from the ravages of time, certainly the building of these magnificent sidereal systems may well engage the attention of the natural philosopher.

20. The foremost geometers of the 18th century, including Lagrange, Laplace and Poisson, were greatly occupied with the problem of the stability of the solar system; and in his historical eulogy on Laplace the penetrating Fourier justly remarks that the researches of geometers prove that the law of gravitation itself operates as a preservative power, and renders all disorder impossible, so that no object is more worthy of the meditation of philosophers than the problem of the stability of these great celestial phenomena.

But if the question of the stability of our single planetary system may so largely absorb the talents of the most illustrious geometers of the age of Herschel, how much more justly may the problem of the stability of clusters, involving many thousands of such systems, claim the attention of the modern geometer, who has witnessed the perfect unfolding of the grand phenomena first discovered by that unrivaled explorer of the heavens?

The grandeur of the study of the origin of the greatest of sidereal systems is worthy of the philosophic penetration of a Herschel! The solution of the dynamical problem presented surpasses the powers of the most titanic geometers, and would demand the inventive genius of a Newton or an Archimedes!

Yet notwithstanding the transcendant character of the problem, and the hopelessness of a rigorous solution in our time, even an imperfect outline of nature's laws may aid the thoughtful astronomer, in penetrating the underlying workings of the sidereal universe, and thus enable him to perceive the great end subserved by the development of the cosmos. If so, he may well rejoice, and exclaim with Ptolemy:

"Though but the being of a day,
When I the planet-paths survey,
My feet the dust despise;
Up to the throne of God I mount
And quaff from an immortal fount
The nectar of the skies."—Transl. by W. B. Smith.

T. J. J. SEE.